with their weights (positive integers), where a k-weighted arc can be interpreted as the set of k parallel arcs. Labels for unity weight are usually omitted. A marking (state) assigns to each place a nonnegative integer. If a marking assigns to place p a nonnegative integer k, we say that p is "marked with" k "tokens." Pictorially, we place k black dots (tokens) in place p. A marking is denoted by M, an m-vector, where m is the total number of places. The pth component of M, denoted by M(p), is the number of tokens in place p.

In the Petri-net modeling where the concept of conditions and events are applicable, places represent conditions and transitions represent events. A transition (an event) has a certain number of *input* and *output places* representing the pre-conditions and post-conditions of the event, respectively. The presence of a token in a place is interpreted as holding the truth of the condition associated with the place. In another interpretation, k tokens are put in a place to indicate that k data items or resources are available. Some typical interpretations of transitions and their input places and output places are shown in Table 1. A formal definition of a Petri net is given in Table 2.

The behavior of many systems can be described in terms of system states and their changes. In order to simulate the dynamic behavior of a system, a state or marking M in a Petri net is changed to M' according to the following *transition (firing) rule:*

1. A transition t is said to be *enabled* if each input place p of t is marked with at least w(p, t) tokens, where w(p, t) is the weight of the arc from p to t, i.e., t is enabled if

$$M(p) \geq w(p, t) \text{ for all } p. \qquad (1\text{-}1)$$

2. An enabled transition may or may not fire (depending on whether or not the event actually takes place).

3. A firing of an enabled transition t removes w(p, t) tokens from each input place p of t, and adds w(t, p) tokens to each output place p of t, where w(t, p) is the weight of the arc from t to p, i.e., the resulting marking M' is given by

$$M'(p) = M(p) - w(p, t) + w(t, p). \qquad (1\text{-}2)$$