

- (a) A finite-capacity net (N, M_0) . (b) The net (N', M'_0) after the transformation.
 (c) The reachability graph for the net (N, M_0) shown in (a).

Step 1: Add a complementary place p' for each place p , where the initial marking of p' is given by $M'_0(p') = K(p) - M_0(p)$.

Step 2: Between each transition t and some complementary places p' , draw new arcs (t, p') or (p', t) where $w(t, p') = w(p, t)$ and $w(p', t) = w(t, p)$, so that the sum of tokens in place p and its complementary place p' equals its capacity $K(p)$ for each place p , before and after firing the transition t .

Example 1.2. Let us apply the strict transition rule to the finite capacity net (N, M_0) shown in Fig. 1.2(a). At the initial marking $M_0 = (1\ 0)$, the only enabled transition is t_1 . After firing t_1 , we have $M_1 = (2\ 0)$, where only t_2 and t_3 are enabled. M_1 changes to $M_2 = (0\ 0)$ after firing t_2 , or to $M_3 = (0\ 1)$ after firing t_3 . Continuing this process, it is easy to draw the (reachability) graph shown in Fig. 1.2(c), which shows all possible markings and all possible firings at each marking. Now, let us see how the net (N, M_0) shown in Fig. 1.2(a) is transformed by the complementary-place transformation into the net (N', M'_0) shown in Fig. 1.2(b). The first step is to add the two complementary places p_1' and p_2' with their initial markings, $M'_0(p_1') = K(p_1) - M_0(p_1) = 2 - 1 = 1$, and $M'_0(p_2') = K(p_2) - M_0(p_2) = 1 - 0 = 1$. The next step is to add new arcs between each transition t and some complementary places, so as to keep the sum of tokens in each pair of places p_i and p_i' the same and equal to $K(p_i)$, $i = 1, 2$, before and after firing t . For example, since $w(t_1, p_1) = 1$, we have $w(p_1', t_1) = 1$. Similarly, $w(t_3, p_1') = w(p_1, t_3) = 2$ and $w(p_2', t_3) = w(t_3, p_2) = 1$, since firing t_3 removes two tokens from p_1 and adds one token in p_2 (we draw the two-weight arc from t_3 to p_1' and the unit-weight arc from p_2' to t_3). Likewise, two additional arcs (t_2, p_1') and (t_4, p_2') are drawn to obtain the net (N', M'_0) shown in Fig. 1.2(b). In a similar manner, as illustrated for (N, M_0) , it is easy to