a net. This leads to a submarking reachability problem which is the problem of finding if $M_n' \in R(M_0)$, where $M_n'$ is any marking whose restriction to a given subset of places agrees with that of a given marking $M_n$. It has been shown that the reachability problem is decidable [174, 175] although it takes at least exponential space (and time) to verify in the general case [275]. However, the equality problem [138, 176, 177] is undecidable, i.e., there is no algorithm for determining if $L(N, M_0) = L(N', M_0')$ for any two Petri nets N and N′.

**Exercise 3.1**. For the Petri net shown in Fig. 3.1, find $L(N, M_0)$ and $R(N, M_0)$.

**Answer:** $L(N, M_0)$ = { < $t_1$ >, < $t_2$ >, < $t_2$, $t_4$ >, < $t_2$, $t_4$, $t_5$ >, < $t_2$, $t_4$, $t_5$, $t_1$>, < $t_2$, $t_4$, $t_5$, $t_2$>, <$t_2$, $t_4$, $t_5$,$t_1$,$t_3$> }. $R(N, M_0)$ = { (1 0 0 1 0 0) = $M_0$, (0 1 0 1 0 0), (0 0 0 1 1 0), (0 0 1 0 0 1), (1 0 1 0 0 0), (0 1 1 0 0 0), (0 0 0 1 0 0), (0 0 1 0 1 0) }.

## 3.2 Boundedness

A Petri net $(N, M_0)$ is said to be *k-bounded* or simply *bounded* if the number of tokens in each place does not exceed a finite number k for any marking reachable from $M_0$, i.e., $M(p) \leq k$ for every place p and for every marking $M \in R(M_0)$. A Petri net $(N, M_0)$ is said to be *safe* if it is 1-bounded. Places in a Petri net are often used to represent buffers and registers for storing intermediate data. By verifying that the net is bounded or safe, it is guaranteed that there will be no overflows in the buffers or registers, no matter what firing sequence is taken.

**Example 3.1**: The nets shown in Figs. 1.2(b), 2.1, 2.4 and 2.8 are all bounded; in particular, the net in Fig. 1.2(b) is 2-bounded, and the rest of the nets are safe.

## 3.3 Liveness