

"fair" control such as the one represented by the loop,  $p_4 \rightarrow t_1 \rightarrow p_5 \rightarrow t_2 \rightarrow p_4$  with  $k$  tokens in  $p_4$ , as is shown in Fig. 3.5(b), it is easy to verify that the synchronic distance between  $t_1$  and  $t_2$  is now changed to  $d_{12} = k = 2 < \infty$ . With this control, it can be seen that one transition can fire at most  $k$  times if the other transition does not fire. This idea of making transition firings bounded leads to a method of defining a "fairness" property as discussed next.

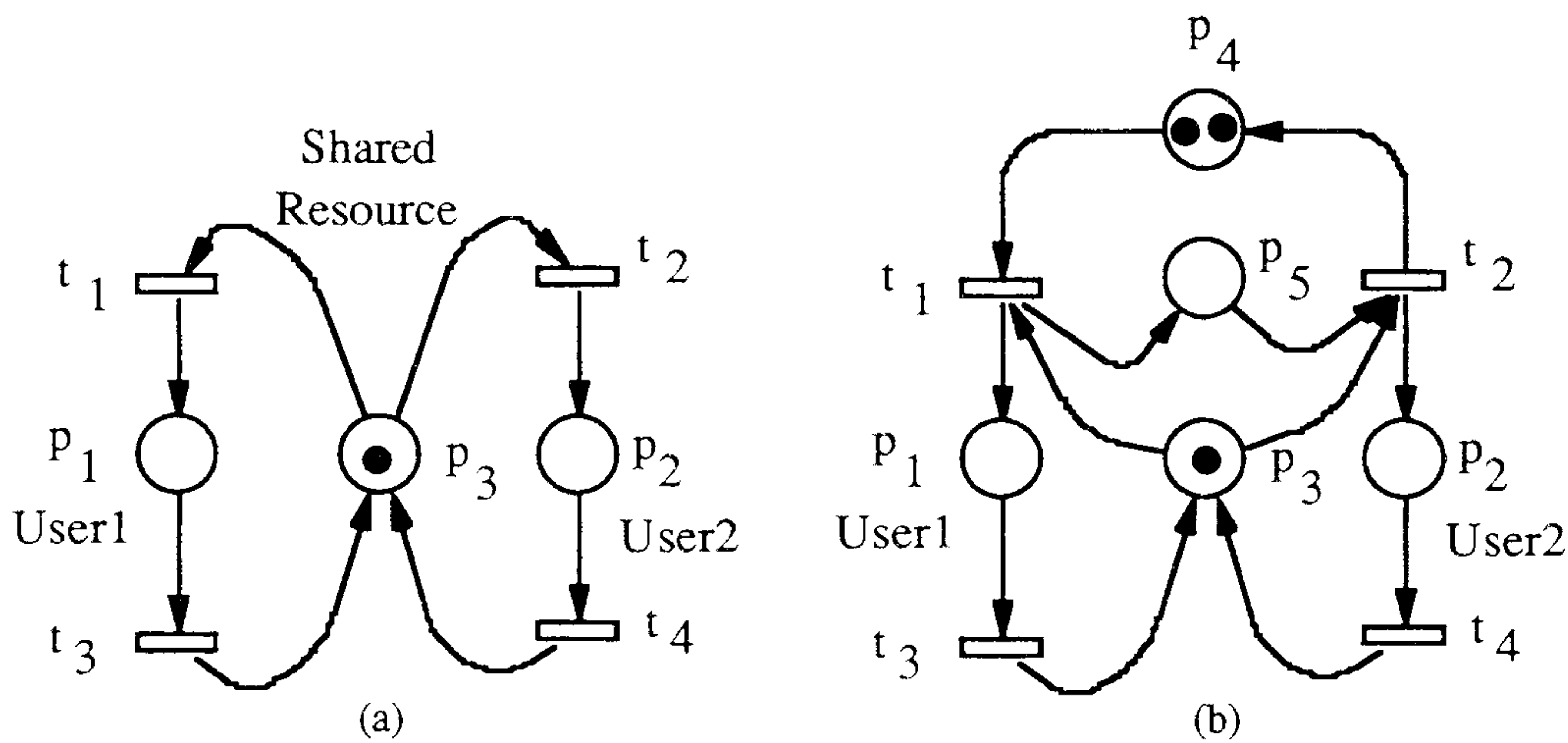


Fig. 3.5 (a) Petri net model of a simple resource sharing system without regulation;  
(b) with regulation or "fairness" control.

### 3.8 Fairness

Many different notions of fairness have been proposed in the literature of Petri nets. There are two types of fairness concepts: one is concerned with boundedness of transition firings and the other with firing sequences (languages). The first type includes B-fair relations and B-fair nets. Two transitions  $t_1$  and  $t_2$  are said to be in a *bounded-fair* (or *B-fair*) *relation* if the maximum number of times that either one can fire while the other is not firing is bounded. A Petri net  $(N, M_0)$  is said to be a *B-fair net* if every pair of transitions in the net are in a B-fair relation. Unconditional (or global) fairness belongs to the second type of fairness concepts, where it is stipulated that all finite firing sequences are fair in all definitions. Thus we only need to state whether an infinite sequence is fair or not. An infinite firing sequence  $\sigma$  is said to be *unconditionally (globally) fair* if every transition in