The concept of liveness is closely related to the complete absence of deadlocks in operating systems. A Petri net (N, M_0) is said to be *live* (or equivalently M_0 is said to be a *live marking* for N) if, no matter what marking has been reached from M_0, it is possible to ultimately fire any transition of the net by progressing through some further firing sequence. This means that a live Petri net guarantees deadlock-free operation, no matter what firing sequence is chosen. Examples of live Petri nets are shown in Figs. 2.1, 2.4 and 2.8. On the other hand, the Petri nets shown in Figs. 3.1 and 3.2 are not live. These nets are not live since no transitions can fire if t_1 fires first.

Liveness is an ideal property for many systems. However, it is impractical and too costly to verify this strong property for some systems such as the operating system of a large computer. Thus, we relax the liveness condition and define different levels of liveness as follows [8, 178]. A transition t in a Petri net (N, M_0) is said to be:

0) *dead (L0-live)* if t can never be fired in any firing sequence in $L(M_0)$.

1) *L1-live (potentially firable)* if t can be fired at least once in some firing sequence in $L(M_0)$.

2) *L2-live* if, given any positive integer k, t can be fired at least k times in some firing sequence in $L(M_0)$.

3) *L3-live* if t appears infinitely often in some firing sequence in $L(M_0)$.

4) *L4-live or live* if t is L1-live for every marking M in $R(M_0)$.

A Petri net (N, M_0) is said to be *Lk-live* if every transition in the net is Lk-live, k = 0, 1, 2, 3, 4. L4-liveness is the strongest and corresponds to the liveness defined earlier. It is easy to see the following implications: L4-liveness => L3-liveness => L2-liveness => L1-liveness, where => means "implies". We say that a transition is *strictly Lk-live* if it is Lk-live but not L(k+1)-live, k = 1, 2, 3.

Example 3.2. The Petri net shown in Fig. 3.1 is strictly L1-live since each transition can be fired exactly once in the order of t_2, t_4, t_5, t_1 and t_3. The transitions t_0,