Theorem 4.1 If M_d is reachable from M_0 in a Petri net (N, M_0), then $B_f \Delta M = 0$, where $\Delta M = M_d - M_0$ and B_f is given by (4-8).

The contrapositive of Theorem 4.1 provides the following sufficient condition for nonreachability.

Corollary 4.1 In a Petri net (N, M_0), a marking M_d is not reachable from M_0 ($\neq M_d$) if their difference is a linear combination of the row vectors of B_f, that is,

$$\Delta M = B_f^T z \tag{4-10}$$

where z is a nonzero $\mu \times 1$ column vector.

Proof: If (4-10) holds, then $B_f \Delta M = B_f B_f^T z \neq 0$, since $z \neq 0$ and $B_f B_f^T$ is a $\mu \times \mu$ nonsingular matrix (because the rank of B_f is $\mu = m - r$). Therefore, by Theorem 4.1, M_d is not reachable from M_0.

Example 4.4 For the Petri net shown in Fig. 4.5, the state equation (4-3) is illustrated below, where the transition t_3 fires to result in the marking $M_1 = (3\ \ 0\ \ 0\ \ 2)^T$ from $M_0 = (2\ \ 0\ \ 1\ \ 0)^T$:

$$\begin{bmatrix} 3 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Using the matrix B_f found in Exercise 4.3, it is easy to verify Theorem 4.1, i.e.,

$B_f \Delta M = 0$ holds for $\Delta M = M_1 - M_0 = (1\ \ 0\ \ -1\ \ 2)^T$:

$$\begin{bmatrix} 1 & 0 & 2 & 1/2 \\ 0 & 1 & -1 & -1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

To verify Corollary 4.1, consider the marking $M_d = (3\ \ 1\ \ 2\ \ 0)^T$. This marking is not reachable, since it can be expressed as $M_d = M_0 + B_f^T z$, where $z = (1\ \ 1)^T \neq 0$.