An integer solution x of the homogeneous equation ($\Delta M = 0$ in (4-5)),

$$A^T x = 0 \qquad (4\text{-}11)$$

is called a *T-invariant*, and an integer solution y of the transposed homogeneous equation $Ay = 0$ is called an *S-invariant*. These invariants which we will discuss in Chapter 7 provide powerful tools for studying structural properties of Petri nets.

4.3 Simple Reduction Rules for Analysis

To facilitate the analysis of a large system, we often reduce the system model to a simpler one, while preserving the system properties to be analyzed. Conversely, techniques to transform an abstracted model into a more refined model in a hierarchical manner can be used for synthesis. There exist many transformation techniques for Petri nets. In this section, we present only the simplest transformations, which can be used for analyzing liveness, safeness and boundedness. Several transformation rules for marked graphs will be discussed in Chapter 8.

It is not difficult to see that the following six operations [179, 203] preserve the properties of liveness, safeness, and boundedness. That is, let (N, M_0) and (N', M_0') be the Petri nets before and after one of the following transformations. Then (N', M_0') is live, safe or bounded iff (N, M_0) is live, safe or bounded, respectively.