4.2.1 Incidence Matrix

For a Petri net N with n transitions and m places, the *incidence matrix* $A = [\, a_{ij} \,]$ is an $n \times m$ matrix of integers and its typical entry is given by

$$a_{ij} = a_{ij}^{+} - a_{ij}^{-} \qquad (4\text{-}1)$$

where $a_{ij}^{+} = w(i, j)$ is the weight of the arc from transition i to its output place j and $a_{ij}^{-} = w(j, i)$ is the weight of the arc to transition i from its input place j. The two $n \times m$ matrices $A^{+} = [\, a_{ij}^{+} \,] = F$ and $A^{-} = [\, a_{ij}^{-} \,] = B$ are called the *forward* and *backward incidence matrices*, respectively. From (4-1), we have $A = A^{+} - A^{-} = F - B$. Note that if there were a self-loop at transition i with place j such that $a_{ij}^{+} = a_{ij}^{-}$, then we have $a_{ij} = 0$, which is the same as a case where there were no arcs between transition i and place j. This is the reason that we have to assume the net is pure whenever we use the incidence matrix. Some authors prefer using $C = A^{T}$, the transposed matrix of A. In this book, we use A instead of its transpose C because A reduces to the well-known incidence matrix of a directed graph in the case of marked graphs, a subclass of Petri nets.

It is easy to see from the transition rule described in Chapter 1 that a_{ij}^{-}, a_{ij}^{+} and a_{ij}, respectively, represent the number of tokens removed, added, and changed in place j, when transition i fires once. Transition i is enabled at a marking M iff

$$a_{ij}^{-} \le M(j), \quad j = 1, 2, 3, \ldots, m. \qquad (4\text{-}2)$$

Exercise 4.2 Write the incidence matrices A, A^{+} and A^{-} for the Petri net shown in Fig. 4.5 and verify (4-2).

Answer: The incidence matrix A, A^{+} and A^{-} are, respectively, given by: