$$A = \begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \end{array} \begin{array}{cccc} p_1 & p_2 & p_3 & p_4 \\ \hline -2 & 1 & 1 & 0 \\ 1 & -1 & 0 & -2 \\ 1 & 0 & -1 & 2 \end{array} \quad A^+ = \begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \end{array} \begin{array}{cccc} p_1 & p_2 & p_3 & p_4 \\ \hline 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{array} \quad A^- = \begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \end{array} \begin{array}{cccc} p_1 & p_2 & p_3 & p_4 \\ \hline 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}$$

Using (4-2), we see that: 1) t_1 is enabled since $a^-_{11} = 2 \le M(1) = 2$, $a^-_{12} = 0 \le M(2) = 0$, $a^-_{13} = 0 \le M(3) = 1$, and $a^-_{14} = 0 \le M(4) = 0$; 2) t_2 is not enabled since $a^-_{22} = 1 > M(2) = 0$; 3) t_3 is enabled since $a^-_{31} = 0 \le M(1) = 2$, $a^-_{32} = 0 \le M(2) = 0$, $a^-_{33} = 1 \le M(3) = 1$, and $a^-_{34} = 0 \le M(4) = 0$, where the marking $M = (2\ 0\ 1\ 0)$ is the initial marking shown in the net.

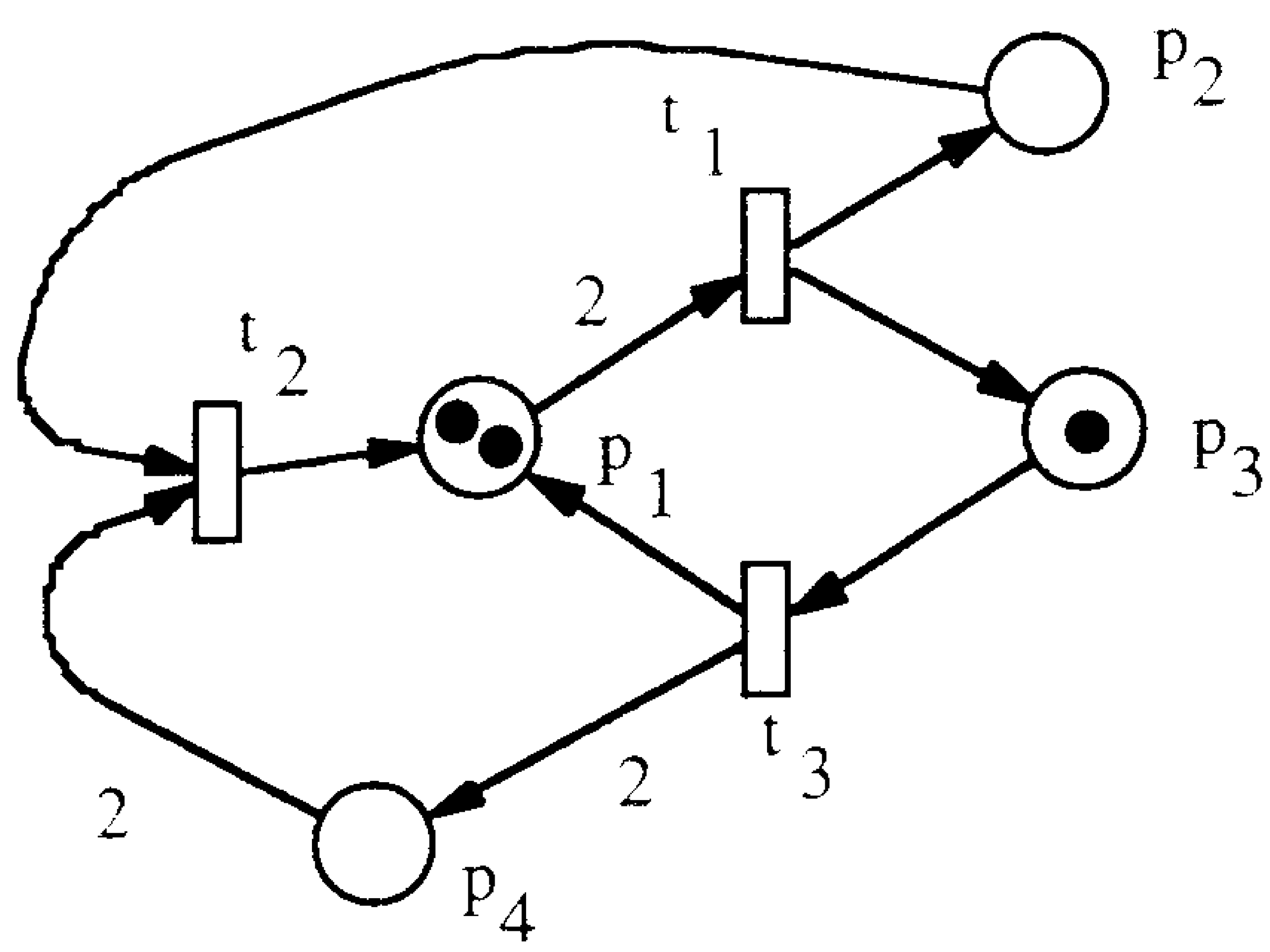

Fig. 4.5. A Petri net used in Exercises 4.2 & 4.3 and Example 4.4

4.2.2 State Equation

In writing matrix equations, we write a marking M_k as an $m \times 1$ column vector. The jth entry of M_k denotes the number of tokens in place j immediately after the kth firing in some firing sequence. The kth firing or control vector u_k is an $n \times 1$ column vector of $(n - 1)$ 0's and one non-zero entry, a 1 in the ith position indicating that transition i fires at the kth firing. Since the ith row of the incidence matrix A denotes the