change of the marking as the result of firing transition i, we can write the following state equation for a Petri net, which was originally proposed by Murata [198]:

$$M_k = M_{k-1} + A^T u_k, \; k = 1, 2, 3, \ldots \qquad (4\text{-}3)$$

4.2.3 Necessary Condition for Reachability in Unrestricted Nets

Suppose that a destination marking M_d is reachable from M_0 through a firing sequence $\{u_1, u_2, \ldots, u_d\}$. Writing the state equation (4-3) for i = 1, 2, ..., d and summing them, we obtain

$$M_d = M_0 + A^T \sum_{k=1}^{d} u_k \qquad (4\text{-}4)$$

which can be rewritten as

$$A^T x = \Delta M \qquad (4\text{-}5)$$

where $\Delta M = M_d - M_0$ and $x = \sum_{k=1}^{d} u_k$. Here x is an $n \times 1$ column vector of nonnegative integers and is called the *firing count vector*. The ith entry of x denotes the number of times that transition i must fire to transform M_0 to M_d. It is well known [199] that a set of linear algebraic equations (4-5) has a solution x over the rational field iff ΔM is orthogonal to every solution y of its transposed homogeneous system,

$$Ay = 0 \qquad (4\text{-}6)$$

Let r be the rank of A, and partition A in the following form,

$$A = \begin{bmatrix} \overset{\overset{m-r}{\longleftrightarrow}}{A_{11}} & \overset{\overset{r}{\longleftrightarrow}}{A_{12}} \\ A_{21} & A_{22} \end{bmatrix} \qquad (4\text{-}7)$$

where A_{12} is a nonsingular square matrix of order r. A set of $(m - r)$ linearly independent solutions y for Eq. (4-6) can be given as the $(m - r)$ rows of the following $(m - r) \times m$ matrix B_f.